MATH1010 Assignment 5

Suggested Solution

1. (a)

$$\int \frac{x}{\sqrt{1-x^2}}\,dx \qquad \text{Put} \quad x = \sin y$$

$$= \int \frac{\sin y}{\sqrt{1-\sin^2 y}} \cos y\,dy \qquad dx = \cos y\,dy$$

$$= \int \frac{\sin y}{\cos y} \cos y\,dy$$

$$= -\cos y + C$$

$$= -\sqrt{1-x^2} + C$$

(b)

$$\int \frac{x}{(1+x^2)^2}\,dx \qquad \text{Put} \quad y = 1 + x^2$$

$$= \int \frac{1}{2y^2}\,dy \qquad dy = 2x\,dx$$

$$= -\frac{1}{2y} + C$$

$$= -\frac{1}{2(1+x^2)} + C$$

(c)

$$\int \frac{e^x}{2+e^x}\,dx \qquad \text{Put} \quad y = e^x$$

$$= \int \frac{1}{2+y}\,dy \qquad dy = e^x\,dx$$

$$= \ln(2+y) + C$$

$$= \ln(2+e^x) + C$$

(d)

$$\int x\sqrt{x-1}\,dx \qquad\qquad \text{Put} \quad y = x-1$$
$$= \int (y+1)\sqrt{y}\,dy \qquad\qquad dy = dx$$
$$= \int y^{\frac{3}{2}} + y^{\frac{1}{2}}\,dy$$
$$= \frac{2}{5}y^{\frac{5}{2}} + \frac{2}{3}y^{\frac{3}{2}} + C$$
$$= \frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + C$$

2. (a)

$$\int x\cos(5-x)\,dx$$
$$= -x\sin(5-x) + \int \sin(5-x)\,dx$$
$$= -x\sin(5-x) + \cos(5-x) + C$$

(b)

$$\int x^2 e^{-2x}\,dx$$
$$= -\frac{1}{2}x^2e^{-2x} + \int xe^{-2x}\,dx$$
$$= -\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}\,dx$$
$$= -\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$$

3. (a)

$$\int \sin^5 x\cos x\,dx$$
$$= \int \sin^5 x\,d\sin x$$
$$= \frac{1}{6}\sin^6 x + C$$

(b)

$$\sin 3x \sin 5x = \frac{\cos 2x - \cos 8x}{2}$$
$$\int \sin 3x \sin 5x \, dx$$
$$= \int \frac{\cos 2x - \cos 8x}{2} \, dx$$
$$= \frac{\sin 2x}{4} - \frac{\sin 8x}{16} + C$$

(c)

$$\sin^2 x = (\frac{1 - \cos 2x}{2})$$
$$\int \sin^4 x \, dx$$
$$= \int (\frac{1 - \cos 2x}{2})^2 \, dx$$
$$= \int \frac{1 - 2\cos 2x + \cos^2 2x}{4} \, dx$$
$$= \frac{x}{4} - \frac{\sin 2x}{4} + \int \frac{1 + \cos 4x}{8} \, dx$$
$$= \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C$$

(d)

$$\int \cos^5 x \sin^4 x \, dx$$
$$= \int \cos^4 x \sin^4 x \, d\sin x$$
$$= \int (1 - \sin^2 x)^2 \sin^4 x \, d\sin x$$
$$= \frac{\sin^5 x}{5} - \frac{2\sin^7 x}{7} + \frac{\sin^9 x}{9} + C$$

(e)

$$\begin{aligned}
&\int \sin^2 x \cos^4 x \, dx \\
&= \int \Big(\frac{1-\cos 2x}{2}\Big)\Big(\frac{1+\cos 2x}{2}\Big)^2 dx \\
&= \int \frac{1+\cos 2x - \cos^2 2x - \cos^3 2x}{8} \, dx \\
&= \frac{x}{8} + \frac{\sin 2x}{16} - \int \frac{\cos^2 2x}{8} \, dx - \int \frac{\cos^3 2x}{8} \, dx \\
&= \frac{x}{8} + \frac{\sin 2x}{16} - \int \frac{1+\cos 4x}{16} \, dx - \int \frac{1-\sin^2 2x}{16} \, d\sin 2x \\
&= \frac{x}{16} - \frac{\sin 4x}{64} + \frac{\sin^3 2x}{48} + C
\end{aligned}$$

(f)

$$\begin{aligned}
&\int \tan^5 x \, dx \\
&= \int \tan^3 x(\sec^2 x - 1) \, dx \\
&= \int \tan^3 x \sec^2 x \, dx - \int \tan^3 x \, dx \\
&= \frac{\tan^4 x}{4} - \int \tan x(\sec^2 x - 1) \, dx \\
&= \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \ln|\sec x| + C
\end{aligned}$$

4. (a)

$$\begin{aligned}
&\int \frac{dx}{(1-x^2)^{\frac{3}{2}}} && \text{Put} \quad x = \sin y \\
&= \int \frac{\cos y}{\cos^3 y} \, dy && dx = \cos y \, dy \\
&= \int \sec^2 y \, dy \\
&= \tan y + C \\
&= \frac{x}{\sqrt{1-x^2}} + C
\end{aligned}$$

(b)

$$\int \frac{dx}{(1+x^2)^{\frac{3}{2}}} \qquad \text{Put} \quad x = \tan y$$

$$= \int \frac{\sec^2 y}{(1+\tan^2 y)^{\frac{3}{2}}}\, dy \qquad dx = \sec^2 y\, dy$$

$$= \int \cos y\, dy$$

$$= \sin y + C$$

$$= \frac{x}{\sqrt{1+x^2}} + C$$

(c)

$$\int \frac{x^2}{\sqrt{9-x^2}}\, dx \qquad \text{Put} \quad x = 3\sin y$$

$$= \int \frac{9\sin^2 y}{3\cos y} 3\cos y\, dy \qquad dx = 3\cos y\, dy$$

$$= \int 9\sin^2 y\, dy$$

$$= \frac{9y}{2} - \frac{9\sin 2y}{4} + C$$

$$= \frac{9}{2}\sin^{-1}\frac{x}{3} - \frac{x\sqrt{9-x^2}}{2} + C$$

(d)

$$\int \frac{dx}{x^2\sqrt{x^2+4}} \qquad \text{Put} \quad x = 2\tan y$$

$$= \int \frac{1}{4\sin^2 y \sec y}\, dy \qquad dx = 2\sec^2 y\, dy$$

$$= \int \frac{1}{4\sin^2 y}\, d\sin y$$

$$= -\frac{1}{4\sin y} + C$$

$$= -\frac{\sqrt{4+x^2}}{4x} + C$$

5. (a)

$$\begin{aligned}
&\int \frac{x^3}{3+x}\,dx \\
&= \int x^2 - 3x + 9 - \frac{27}{x+3}\,dx \\
&= \frac{x^3}{3} - \frac{3x^2}{2} + 9x - 27\ln|x+3| + C
\end{aligned}$$

(b)

$$\begin{aligned}
&\int \frac{(1+x)^2}{1+x^2}\,dx \\
&= \int 1 + \frac{2x}{1+x^2}\,dx \\
&= x + \ln|(1+x^2)| + C
\end{aligned}$$

(c)

$$\begin{aligned}
&\int \frac{dx}{x^2+2x-3} \\
&= \int \frac{dx}{(x+3)(x-1)} \\
&= \int -\frac{1}{4(x+3)} + \frac{1}{4(x-1)}\,dx \\
&= -\frac{1}{4}\ln|(x+3)| + \frac{1}{4}\ln|(x-1)| + C
\end{aligned}$$

(d)

$$\begin{aligned}
&\int \frac{4-2x}{(x^2+1)(x-1)^2}\,dx \\
&= \int \frac{2x}{x^2+1} + \frac{1}{x^2+1} - \frac{2}{x-1} + \frac{1}{(x-1)^2}\,dx \\
&= \ln(x^2+1) + \tan^{-1}x - 2\ln|x-1| - \frac{1}{x-1} + C
\end{aligned}$$

6. (a)

$$\begin{aligned}
\int f(x)\,dx &= \int_1^x f(t)\,dt + C \\
&= \begin{cases} 2x^2 - x - 1 + C & x < 1 \\ 6\sqrt{x} - 6 + C & x \geqslant 1 \end{cases}
\end{aligned}$$

(b)

$$\begin{aligned}\int f(x)\,dx &= \int_3^x f(t)\,dt + C \\ &= \begin{cases} \dfrac{x^2}{2} - 3x + \dfrac{9}{2} + C & x \geqslant 3 \\ 3x - \dfrac{x^2}{2} - \dfrac{9}{2} + C & x < 3 \end{cases}\end{aligned}$$

(c)

$$\begin{aligned}\int f(x)\,dx &= \int_0^x f(t)\,dt + C \\ &= \begin{cases} \dfrac{e^{2x}}{2} - \dfrac{1}{2} + C & x < 0 \\ \dfrac{\sin 2x}{4} + \dfrac{x}{2} + C & x \geqslant 0 \end{cases}\end{aligned}$$

(d)

$$\begin{aligned}\int f(x)\,dx &= \int_0^x f(t)\,dt + C \\ &= \begin{cases} \ln(1+x) + C & x \geqslant 0 \\ \ln(1-x) + C & x \leqslant 0 \end{cases}\end{aligned}$$

7. (a)

$$\begin{aligned} I_n &= \int \frac{x^n}{\sqrt{x+a}}\,dx \\ &= 2x^n\sqrt{x+a} - 2\int nx^{n-1}\sqrt{x+a}\,dx \\ &= 2x^n\sqrt{x+a} - 2n\int \frac{x^{n-1}(x+a)}{\sqrt{x+a}}\,dx \\ &= 2x^n\sqrt{x+a} - 2n\int \frac{x^n}{\sqrt{x+a}}\,dx - 2na\int \frac{x^{n-1}}{\sqrt{x+a}}\,dx \\ (2n+1)I_n &= 2x^n\sqrt{x+a} - 2anI_{n-1} \\ I_n &= \frac{2x^n\sqrt{x+a}}{2n+1} - \frac{2an}{2n+1}I_{n-1} \end{aligned}$$

(b)

$$\begin{aligned}
I_n &= \int \frac{1}{\sin^n x}\,dx \\
&= -\frac{\cos x}{\sin^{n+1} x} - (n+1)\int \frac{\cos^2 x}{\sin^{n+2} x}\,dx \\
&= -\frac{\cos x}{\sin^{n+1} x} - (n+1)\int \frac{1}{\sin^{n+2} x}\,dx + (n+1)\int \frac{1}{\sin^n x}\,dx \\
(n+1)I_{n+2} &= -\frac{\cos x}{\sin^{n+1} x} + nI_n \\
I_n &= -\frac{\cos x}{(n-1)\sin^{n-1} x} + \frac{n-2}{n-1}I_{n-2}
\end{aligned}$$

(c)

$$\begin{aligned}
I_n &= \int \cos^n x\,dx \\
&= \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x \sin^2 x\,dx \\
&= \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x\,dx - (n-1)\int \cos^n x\,dx \\
&= \cos^{n-1} x \sin x + (n-1)I_{n-2} - (n-1)I_n \\
I_n &= \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n}I_{n-2}
\end{aligned}$$

(d)

$$\begin{aligned}
I_n &= \int_0^1 x^n\sqrt{1-x}\,dx \\
&= x^n(-\frac{2}{3}(1-x)^{\frac{3}{2}}\Big|_0^1 + \frac{2n}{3}\int_0^1 x^{n-1}\sqrt{1-x}(1-x)\,dx \\
&= \frac{2n}{3}\int_0^1 x^{n-1}\sqrt{1-x}\,dx - \frac{2n}{3}\int_0^1 x^n\sqrt{1-x}\,dx \\
\frac{2n+3}{3}I_n &= \frac{2n}{3}I_{n-1} \\
I_n &= \frac{2n}{2n+3}I_{n-1}
\end{aligned}$$

8. (a) By the answer of 7(c),

$$\begin{aligned}
&\int_0^\pi \cos^6 x\,dx \\
=&\frac{\sin x\cos^5 x}{6}\bigg|_0^\pi + \frac{5}{6}\int_0^\pi \cos^4 x\,dx \\
=&\frac{5}{6}\int_0^\pi \cos^4 x\,dx \\
=&\frac{5}{6}\cdot\frac{3}{4}\int_0^\pi \cos^2 x\,dx \\
=&\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{2}\int_0^\pi 1\,dx \\
=&\frac{5\pi}{16}
\end{aligned}$$

(b) By the answer of 7(d),

$$\begin{aligned}
&\int_0^1 x^4\sqrt{1-x}\,dx \\
=&\frac{8}{11}\int_0^1 x^3\sqrt{1-x}\,dx \\
=&\dots \\
=&\frac{8}{11}\cdot\frac{6}{9}\cdot\frac{4}{7}\cdot\frac{2}{5}\int_0^1 \sqrt{1-x}\,dx \\
=&\frac{8}{11}\cdot\frac{6}{9}\cdot\frac{4}{7}\cdot\frac{2}{5}(-\frac{2}{3}(1-x)^{\frac{3}{2}}\bigg|_0^1 \\
=&\frac{256}{3465}
\end{aligned}$$

9. (a)

$$\int x^2\sqrt{16-x^2}\,dx \qquad\qquad \text{Put} \quad x = 4\sin y$$

$$= \int 256\sin^2 y\cos^2 y\,dy \qquad\qquad dx = 4\cos y\,dy$$

$$= 256\int (\frac{1-\cos 2y}{2})(\frac{1+\cos 2y}{2})\,dy$$

$$= 256\int \frac{1-\cos^2 2y}{4}\,dy$$

$$= 256\int \frac{1}{8} - \frac{\cos 4y}{8}\,dy$$

$$= 256(\frac{y}{8} - \frac{\sin 4y}{32}) + C$$

$$= 32\sin^{-1}(\frac{x}{4}) - 2x\sqrt{16-x^2} + \frac{x^3\sqrt{16-x^2}}{4} + C$$

(b)

$$\int \tan^{-1} x\,dx$$

$$= x\tan^{-1} x - \int \frac{x}{1+x^2}\,dx$$

$$= x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + C$$

(c)

$$\int \frac{1}{e^x + e^{-x}}\,dx \qquad \text{Put} \quad y = e^x$$

$$= \int \frac{e^x}{e^{2x}+1}\,dx \qquad\qquad dy = e^x\,dx$$

$$= \int \frac{1}{y^2}\,dy$$

$$= \tan^{-1} y + C$$

$$= \tan^{-1} e^x + C$$

(d)

$$\begin{aligned}
&\int \sin 5x \cos x \, dx \\
=&\int \frac{\sin 6x + \sin 4x}{2} \, dx \\
=&-\frac{\cos 6x}{12} - \frac{\cos 4x}{8} + C
\end{aligned}$$

(e)

$$\begin{aligned}
&\int x^2 \ln x \, dx \\
=&\frac{x^3 \ln x}{3} - \int \frac{x^3}{3} \frac{1}{x} \, dx \\
=&\frac{x^3 \ln x}{3} - \frac{x^3}{9} + C
\end{aligned}$$

(f)

$$\begin{aligned}
&\int \frac{2x^3 - 4x^2 - x - 3}{x^2 - 2x - 3} \, dx \\
=&\int 2x + \frac{5x - 3}{x^2 - 2x - 3} \, dx \\
=&x^2 \int \frac{5x - 3}{(x-3)(x+1)} \\
=&x^2 + \int \frac{3}{x-3} + \frac{2}{x+1} \, dx \\
=&x^2 + 3 \ln |x - 3| + 2 \ln |x + 1| + C
\end{aligned}$$

(g)

$$\begin{aligned}
&\int \frac{e^{3x}+1}{e^x+1}\,dx \\
=&\int \frac{(e^x+1)(e^{2x}-e^x+1)}{e^x+1}\,dx \\
=&\int e^{2x}-e^x+1\,dx \\
=&\frac{e^{2x}}{2}-e^x+x+C
\end{aligned}$$

(h)

$$\begin{aligned}
&\int \frac{\sin x\cos^3 x}{1+\cos^2 x}\,dx && \text{Put} \quad y=\cos^2 x \\
=&-\frac{1}{2}\int \frac{y}{1+y}\,dy && dy=-2\cos x\sin x\,dx \\
=&-\frac{1}{2}\int 1-\frac{1}{1+y}\,dy \\
=&-\frac{\cos^2 x}{2}+\frac{\ln(1+\cos^2 x)}{2}+C
\end{aligned}$$

(i)

$$\begin{aligned}
&\int \frac{x^2}{(x^2-3x+2)^2}\,dx \\
=&\int \frac{4}{x-1}+\frac{1}{(x-1)^2}-\frac{4}{x-2}+\frac{4}{(x-2)^2}\,dx \\
=&4\ln|x-1|-\frac{1}{x-1}-4\ln|x-2|-\frac{4}{x-2}+C
\end{aligned}$$

(j)

$$\begin{aligned}
&\int \frac{x^2}{\sqrt{9-x^3}}\,dx && \text{Put} \quad y=9-x^3 \\
=&-\frac{1}{3}\int \frac{1}{\sqrt{y}}\,dy && dy=-3x^2\,dx \\
=&-\frac{1}{3}(2\sqrt{y})+C \\
=&-\frac{2}{3}\sqrt{9-x^3}+C
\end{aligned}$$

(k)

$$\int x\sec^2 x\,dx$$
$$=x\tan x - \int \tan x\,dx$$
$$=x\tan x - \ln|\sec x| + C$$

(l)

$$\int \frac{dx}{\cos x\sin^2 x}$$
$$= \int \frac{\cos x}{\cos^2 x\sin^2 x}\,dx \qquad \text{Put} \quad y = \sin x$$
$$= \int \frac{1}{(1-y^2)y^2}\,dy \qquad dy = \cos x\,dx$$
$$= \int \frac{1}{y^2} + \frac{1}{2(1+y)} + \frac{1}{2(1-y)}\,dy$$
$$= -\frac{1}{y} + \frac{1}{2}\ln|1+y| - \frac{1}{2}\ln|1-y| + C$$
$$= -\frac{1}{\sin x} + \frac{\ln|1+\sin x|}{2} - \frac{\ln|1-\sin x|}{2} + C$$

(m)

$$\int \ln(x+\sqrt{1+x^2})\,dx$$
$$=x\ln(x+\sqrt{1+x^2}) - \int x\frac{1+x(1+x^2)^{-\frac{1}{2}}}{x+\sqrt{1+x^2}}\,dx$$
$$=x\ln(x+\sqrt{1+x^2}) - \int \frac{x(\sqrt{1+x^2}+x)}{(x+\sqrt{1+x^2})\sqrt{1+x^2}}\,dx$$
$$=x\ln(x+\sqrt{1+x^2}) - \int \frac{x}{\sqrt{1+x^2}}\,dx$$
$$=x\ln(x+\sqrt{1+x^2}) - \sqrt{1+x^2} + C$$

(n)

$$\begin{aligned}
&\int \frac{1}{x^2}\sin\frac{1}{x}\,dx \quad \text{Put} \quad y=\frac{1}{x}\\
&= -\int \sin y\,dy \qquad\qquad dy = -\frac{1}{x^2}\,dx\\
&= \cos y + C\\
&= \cos\frac{1}{x} + C
\end{aligned}$$

(o)

$$\begin{aligned}
&\int \frac{dx}{(4x^2+1)^{\frac{3}{2}}} \qquad \text{Put} \quad x=\frac{1}{2}\tan y\\
&= \int \frac{\frac{1}{2}\sec^2 y}{(\sec^2 y)^{\frac{3}{2}}}\,dy \qquad\qquad dx=\frac{1}{2}\sec^2 y\,dy\\
&= \frac{1}{2}\int \cos y\,dy\\
&= \frac{1}{2}\sin y + C\\
&= \frac{x}{\sqrt{1+4x^2}} + C
\end{aligned}$$

(p)

$$\begin{aligned}
&\int \frac{x}{\sqrt{x+9}}\,dx\\
&= \int \sqrt{x+9} - \frac{9}{\sqrt{x+9}}\,dx\\
&= \frac{2}{3}(x+9)^{\frac{3}{2}} - 18\sqrt{x+9} + C
\end{aligned}$$

(q)

$$\begin{aligned}
\int e^{2x}\cos 3x\,dx &= \frac{e^{2x}\sin 3x}{3} - \frac{2}{3}\int e^{2x}\sin 3x\,dx\\
&= \frac{e^{2x}\sin 3x}{3} + \frac{2e^{2x}\cos 3x}{9} - \frac{4}{9}\int e^{2x}\cos 3x\,dx\\
\frac{13}{9}\int e^{2x}\cos 3x\,dx &= \frac{e^{2x}\sin 3x}{3} + \frac{2e^{2x}\cos 3x}{9} + C\\
\int e^{2x}\cos 3x\,dx &= \frac{3e^{2x}\sin 3x}{13} + \frac{2e^{2x}\cos 3x}{13} + C
\end{aligned}$$

(r)

$$\int \frac{dx}{\sqrt{x}(1+x)} \qquad \text{Put} \quad y = \sqrt{x}$$

$$= 2\int \frac{dy}{1+y^2} \qquad dy = \frac{1}{2\sqrt{x}}dx$$

$$= 2\tan^{-1} y + C$$

$$= 2\tan^{-1} \sqrt{x} + C$$

(s)

$$\int x\sin^2 x \, dx$$

$$= \int x(\frac{1-\cos 2x}{2}) \, dx$$

$$= \frac{x^2}{4} - \int \frac{x\cos 2x}{2} \, dx$$

$$= \frac{x^2}{4} - \Big(\frac{x\sin 2x}{4} - \int \frac{\sin 2x}{4} \, dx\Big)$$

$$= \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + C$$

(t)

$$\int \cos x \cos 2x \cos 3x \, dx$$

$$= \int \Big(\frac{\cos 4x + \cos 2x}{2}\Big) \cos 2x \, dx$$

$$= \int \frac{\cos 6x + \cos 4x + \cos 2x + 1}{4} \, dx$$

$$= \frac{\sin 6x}{24} + \frac{\sin 4x}{16} + \frac{\sin 2x}{8} + \frac{x}{4} + C$$

(u)

$$\begin{aligned}
&\int x^3(1+3x^2)^{\frac{1}{2}}\,dx\\
=&\frac{x^2(1+3x^2)^{\frac{3}{2}}}{9}-\frac{2}{9}\int x(1+3x^2)^{\frac{3}{2}}\,dx\\
=&\frac{x^2(1+3x^2)^{\frac{3}{2}}}{9}-\frac{2(1+3x^2)^{\frac{5}{2}}}{135}+C
\end{aligned}$$

(v)

$$\begin{aligned}
&\int \frac{x^2+5x+4}{x^4+5x^2+4}\,dx\\
=&\int \frac{x^2+5x+4}{(x^2+4)(x^2+1)}\,dx\\
=&\int \frac{-5x}{3(x^2+4)}+\frac{5x+3}{3(x^2+1)}\,dx\\
=&-\frac{5}{6}\ln(x^2+4)+\int \frac{5}{3}\frac{x}{x^2+1}+\frac{1}{x^2+1}\,dx\\
=&-\frac{5}{6}\ln(x^2+4)+\frac{5}{6}\ln(x^2+1)+\tan^{-1}x+C
\end{aligned}$$

(w)

$$\begin{aligned}
&\int \ln^2(x)\,dx\\
=&x\ln^2(x)-2\int \ln(x)\,dx\\
=&x\ln^2(x)-2(x\ln x-\int x\cdot\frac{1}{x}\,dx)\\
=&x\ln^2(x)-2x\ln x+2x+C
\end{aligned}$$

(x)

$$\int \frac{dx}{1-\cos x} \qquad \text{Put} \quad y = \frac{x}{2}$$

$$= \int \frac{dy}{\sin^2 y} \qquad dy = \frac{dx}{2}$$

$$= -\frac{\cos y}{\sin y} + C$$

$$= -\cot(\frac{x}{2}) + C$$